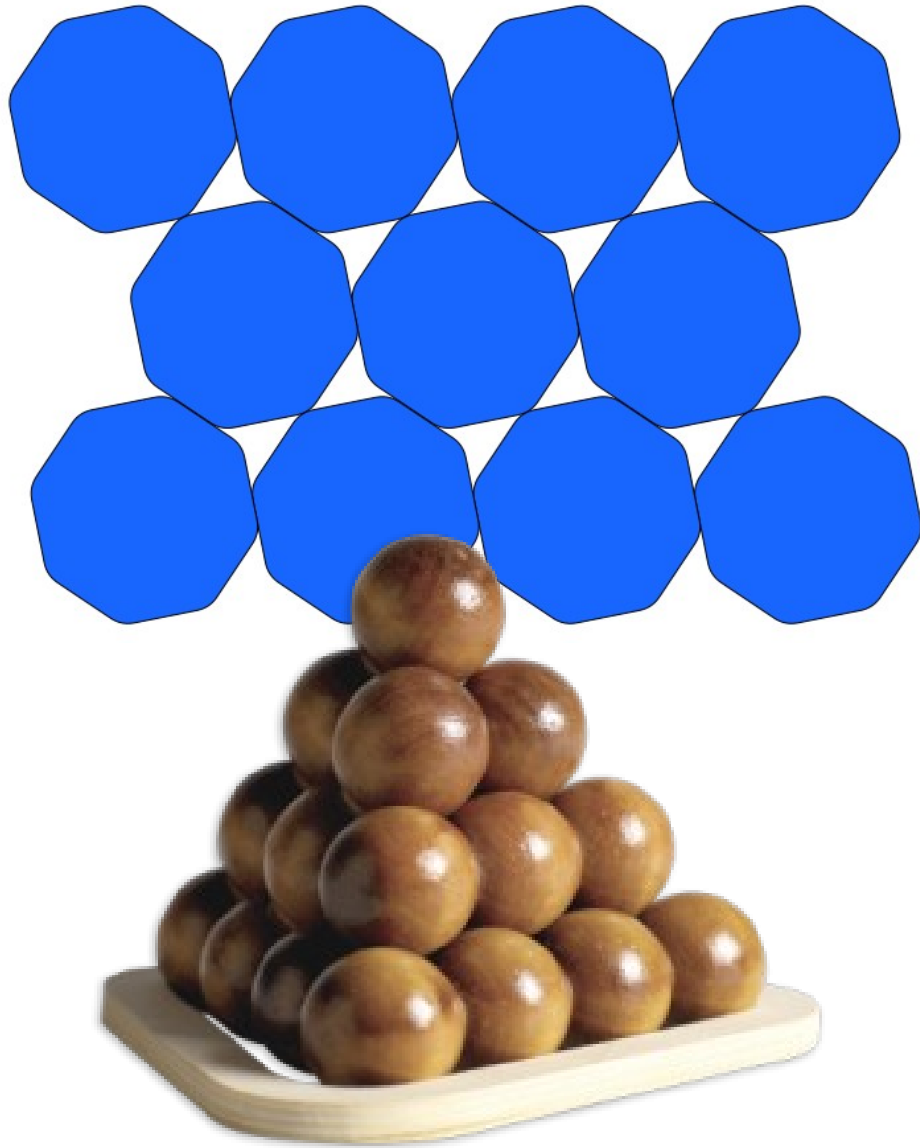
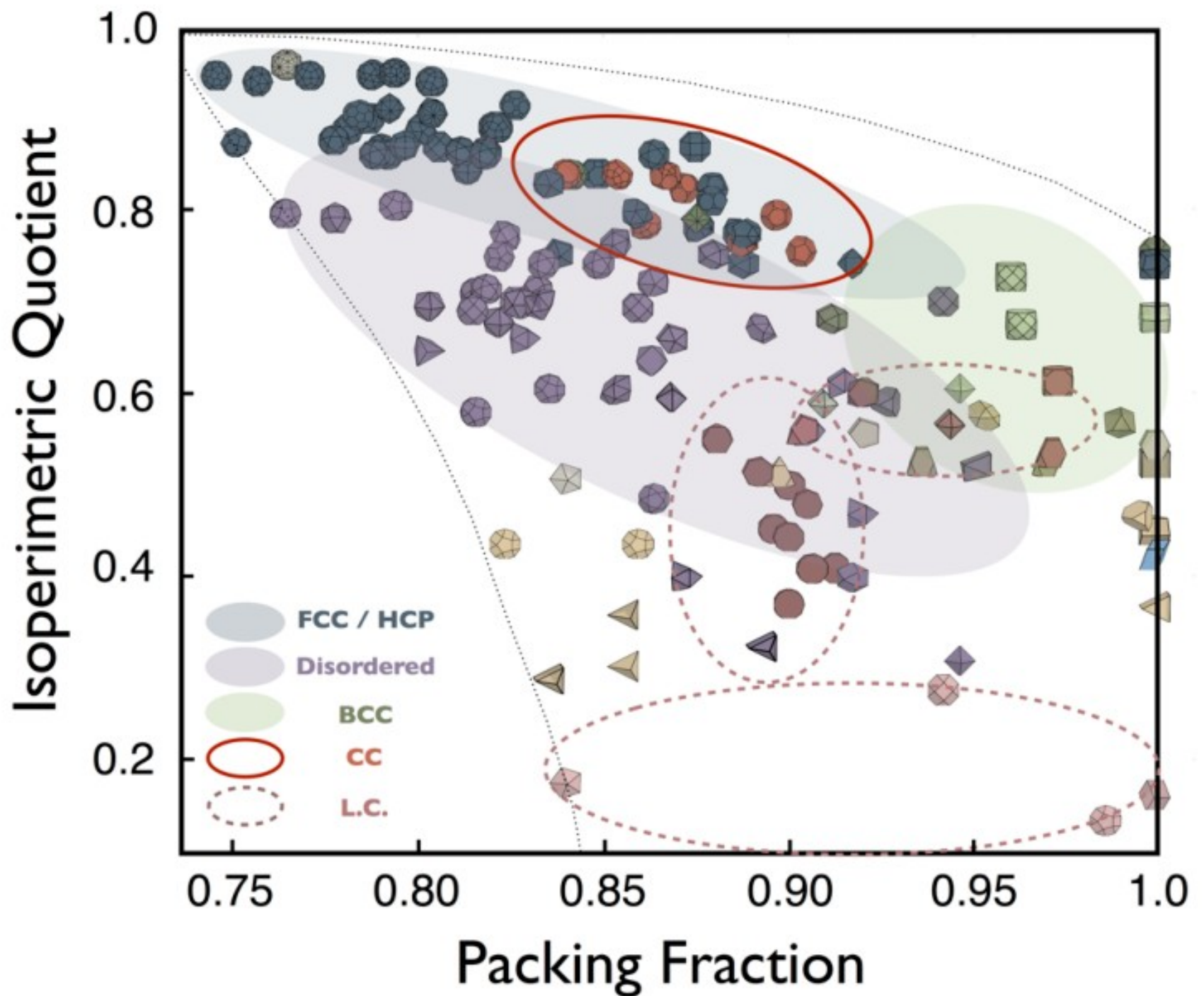


On Ulam's Packing Conjecture



Yoav Kallus
Princeton University
j/w: Fedor Nazarov
Kent University

APS March Meeting, Baltimore, Mar. 18, 2013



Damasceno, Engel & Glotzer (unpublished)

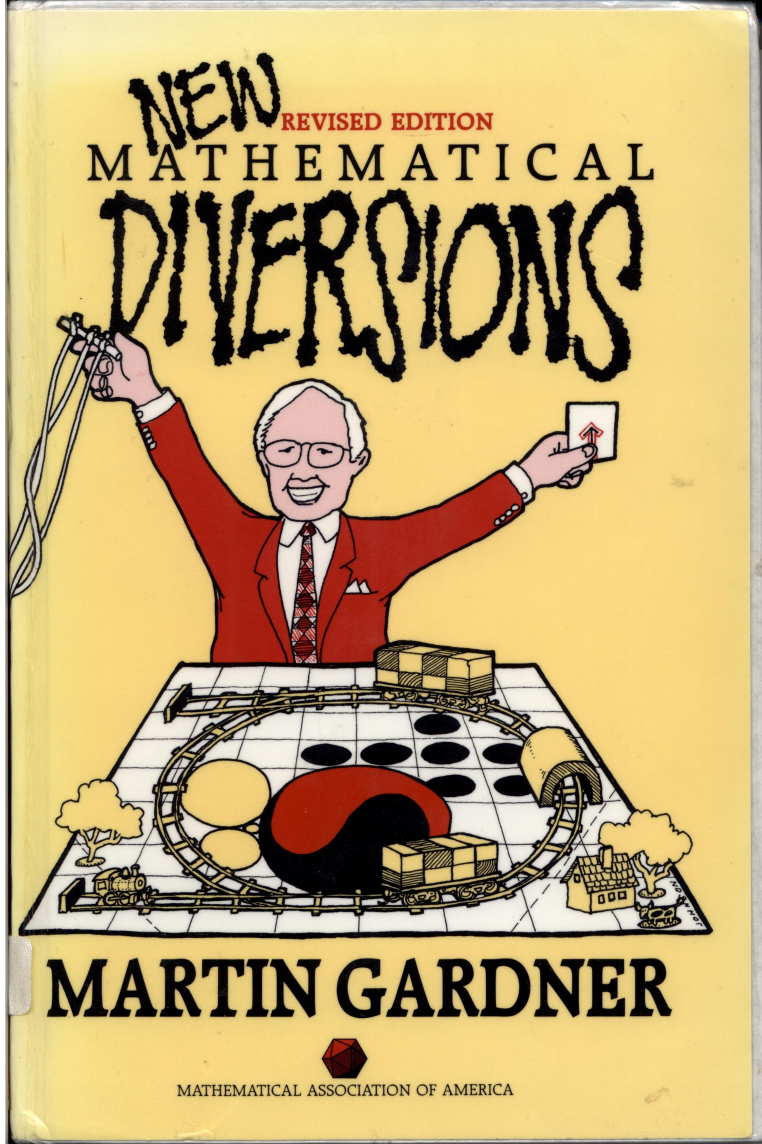
Best packing
shapes are →
trivial



Best packing
shapes are
trivial



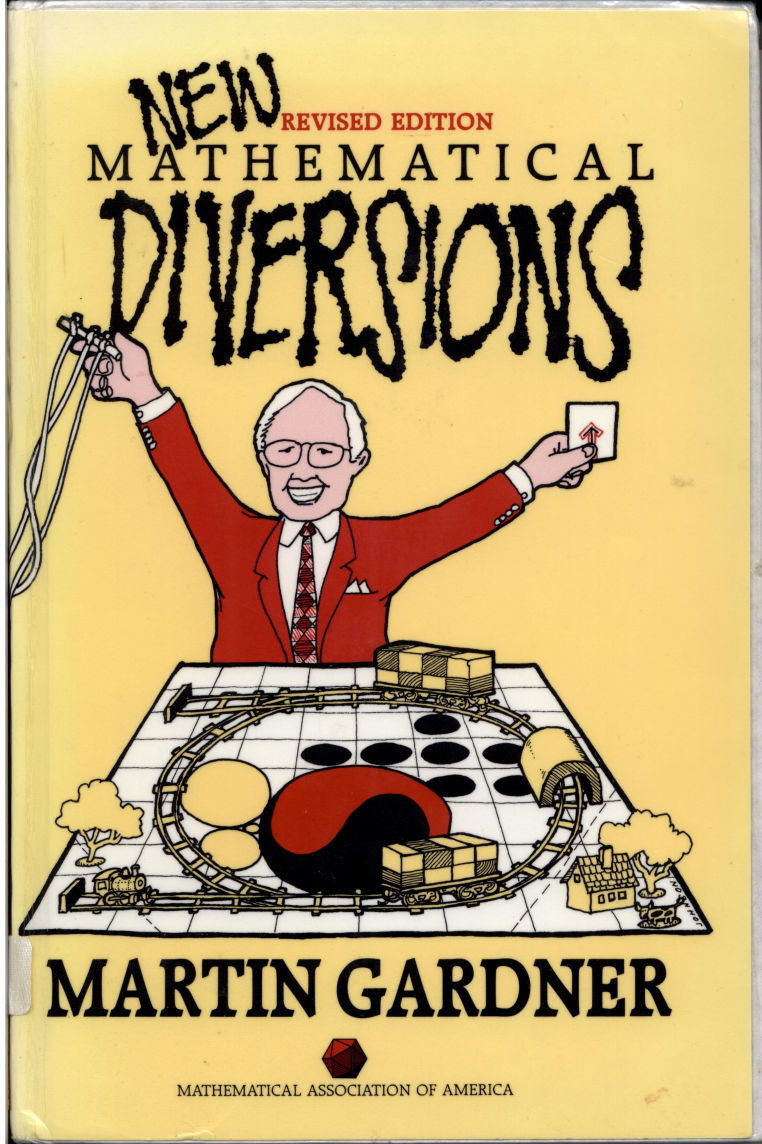
Worst packing
(convex) shape?



Ulam's Conjecture

“Stanislaw Ulam told me in 1972 that he suspected the sphere was the worst case of dense packing of identical convex solids, but that this would be difficult to prove,”

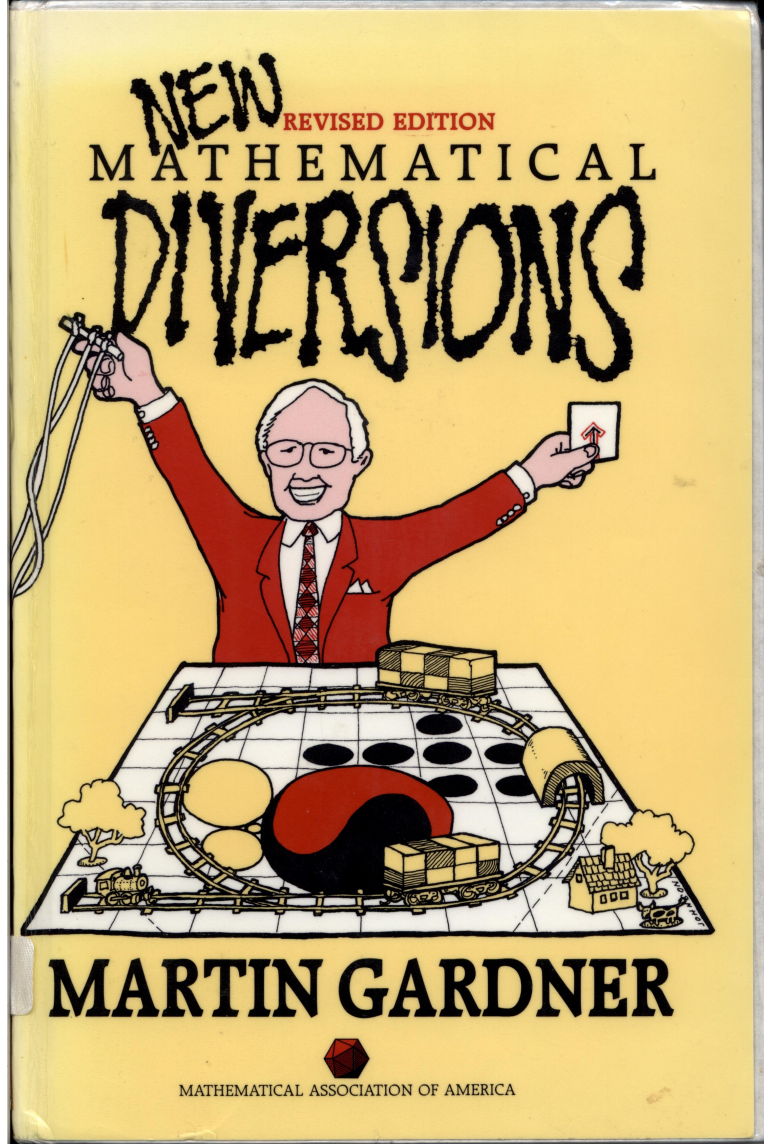
--- 1995 postscript to “Packing Spheres”.



Ulam's "Last" Conjecture

"Stanislaw Ulam told me in 1972 that he suspected the sphere was the worst case of dense packing of identical convex solids, but that this would be difficult to prove,"

--- 1995 postscript to
"Packing Spheres".



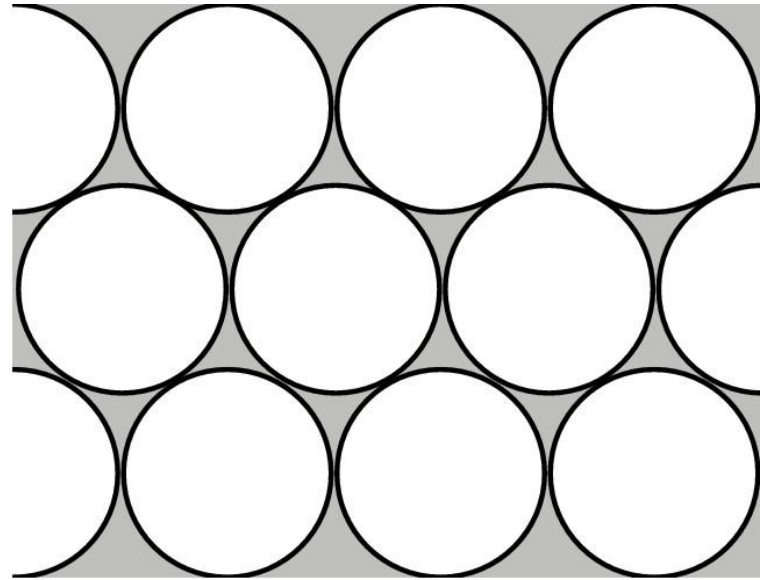
Ulam's “Last” Conjecture

“Stanislaw Ulam told me in 1972 that he suspected the sphere was the worst case of dense packing of identical convex solids, but that this would be difficult to prove,”

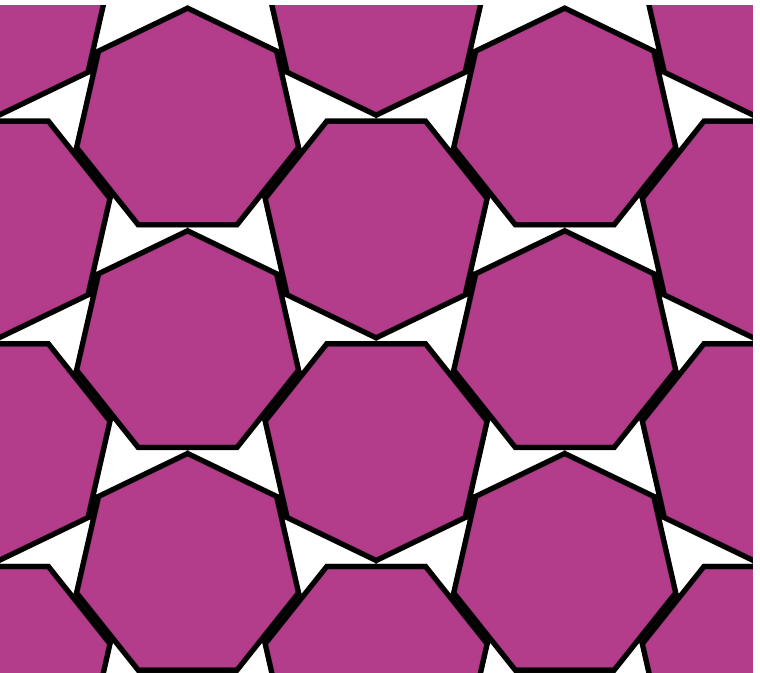
--- 1995 postscript to
“Packing Spheres”.

Naive motivation: sphere is the least free solid (three degrees of freedom vs. six for most solids).

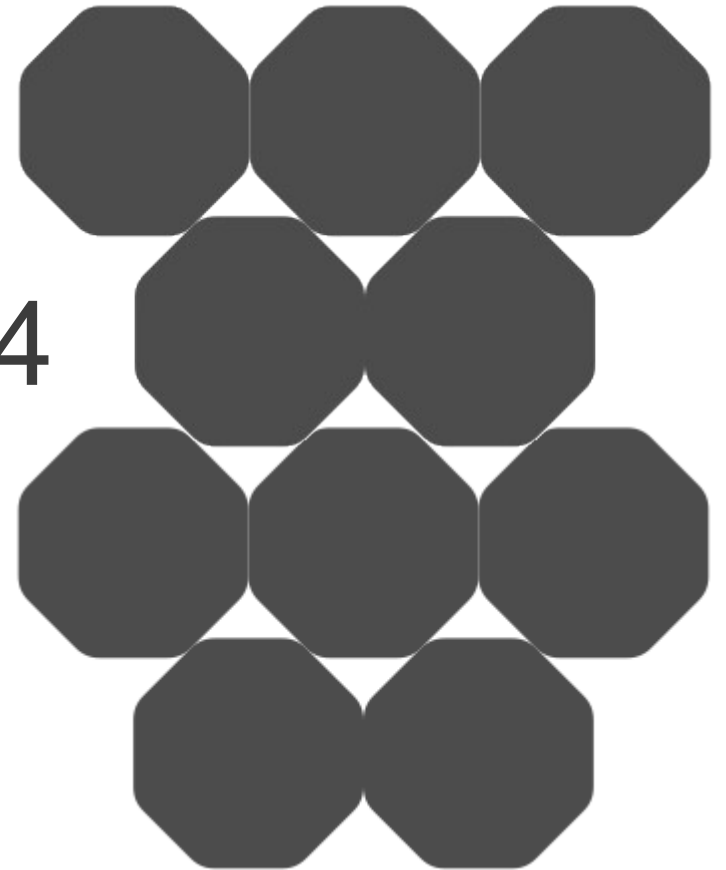
In 2D disks are not worst packing



0.9069

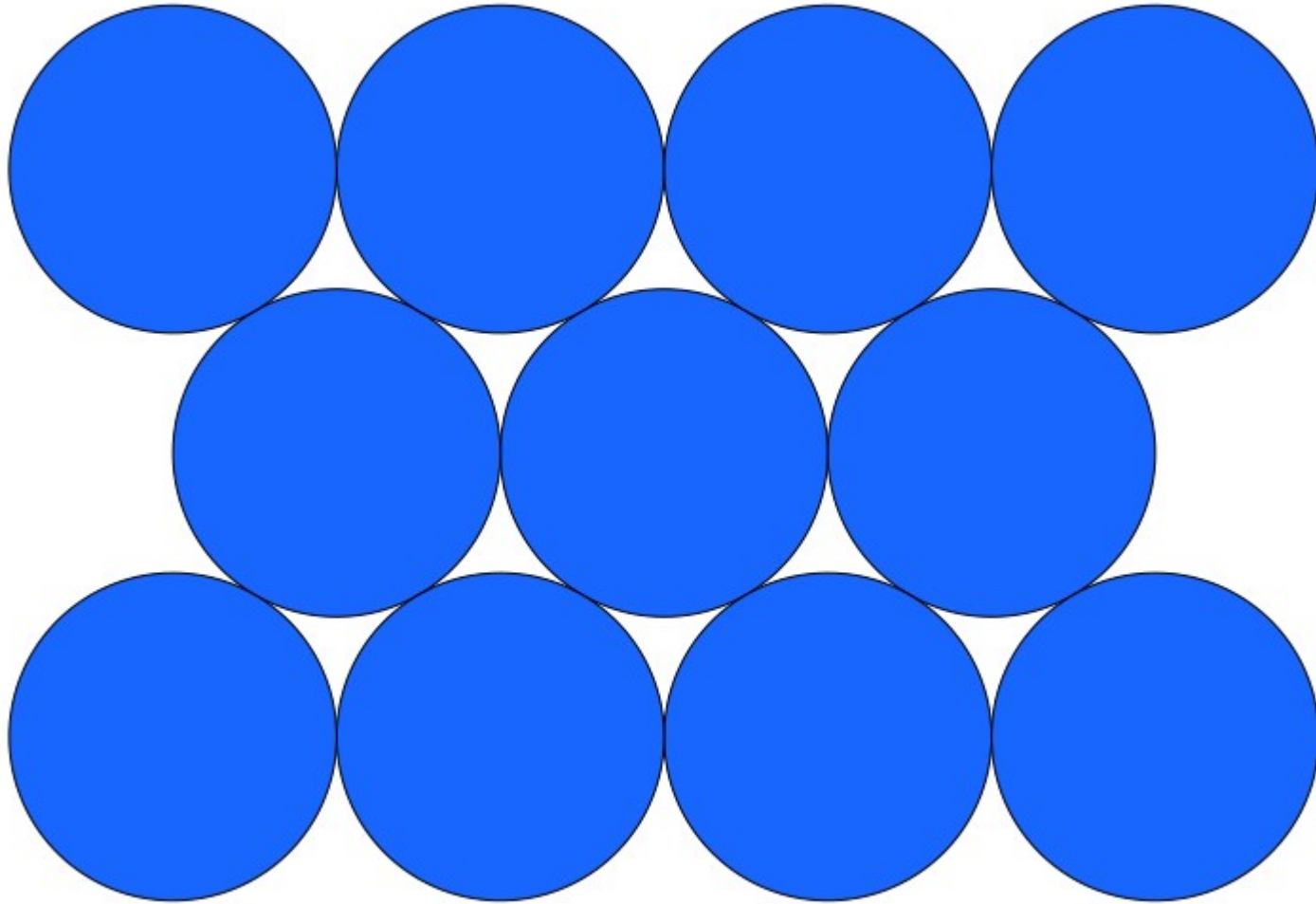


0.8926

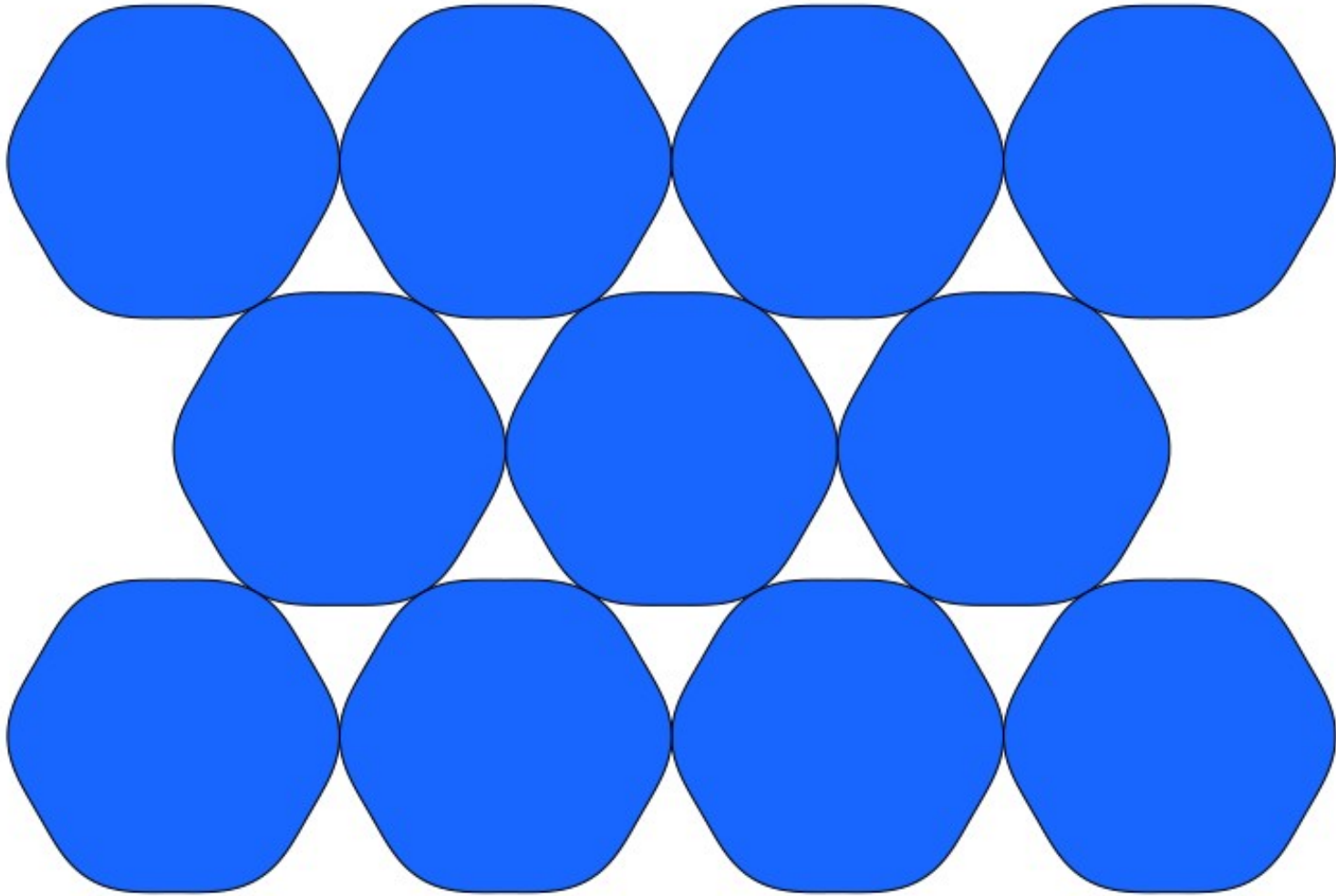


0.9024

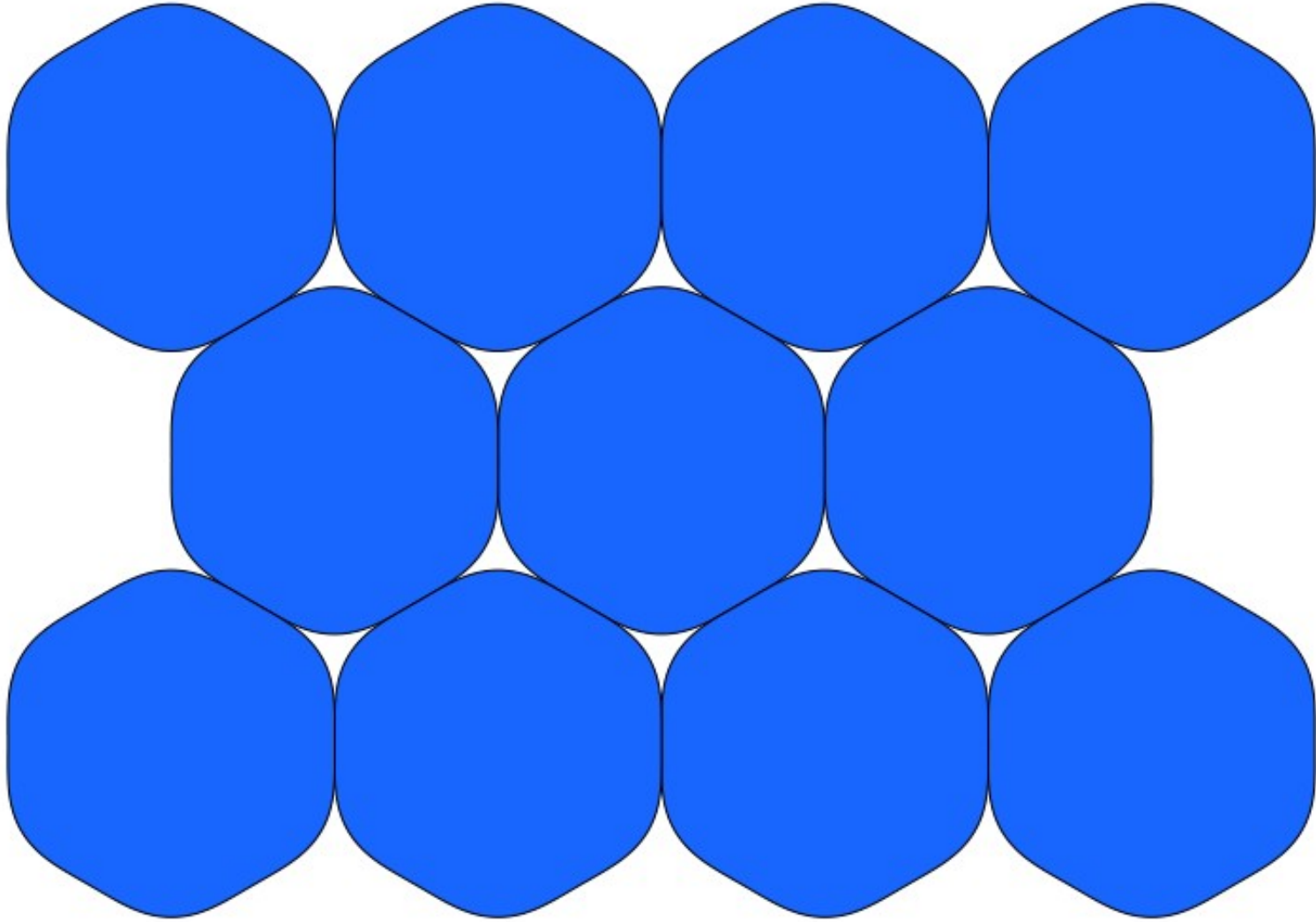
Why can we improve over circles?



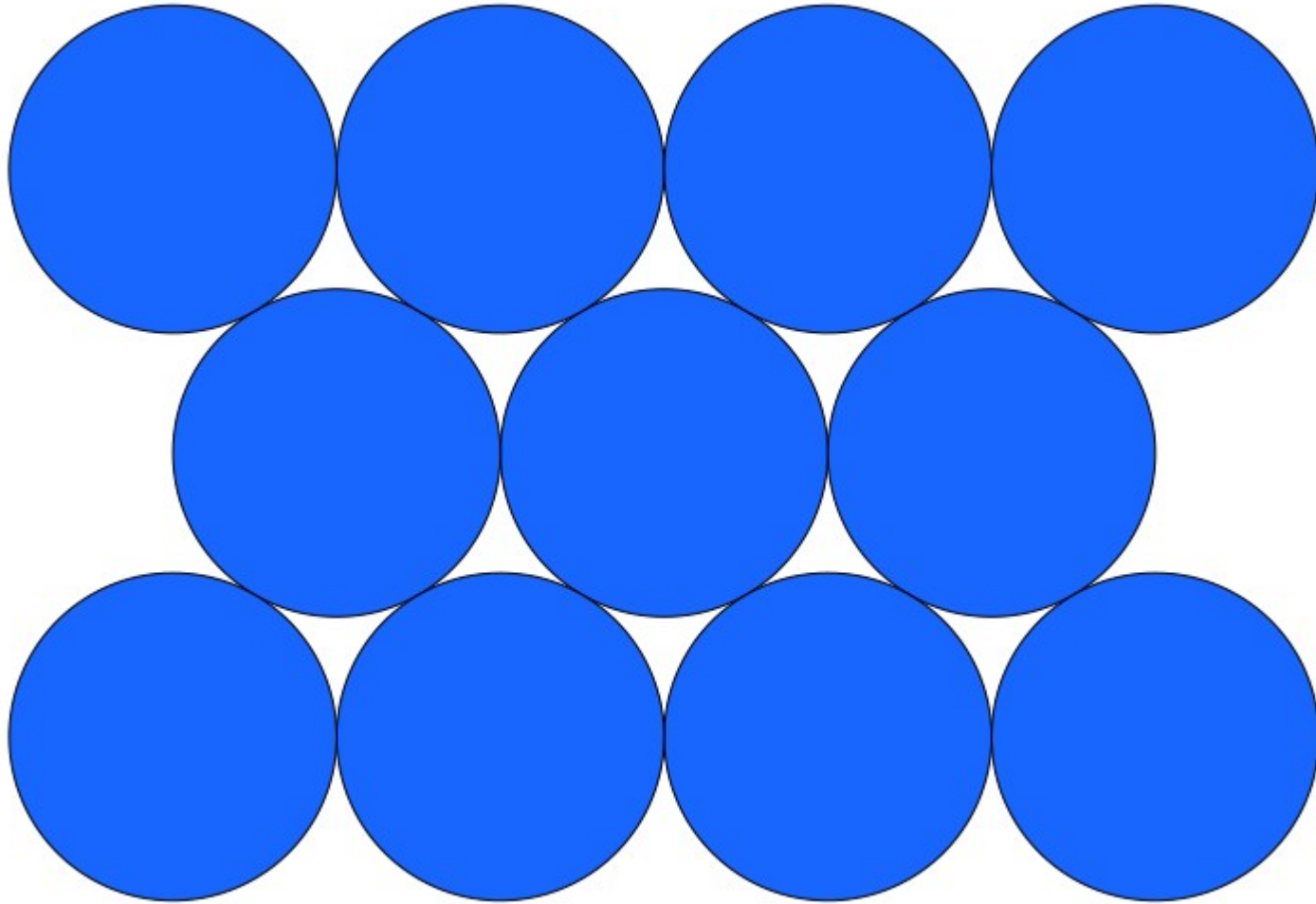
Why can we improve over circles?



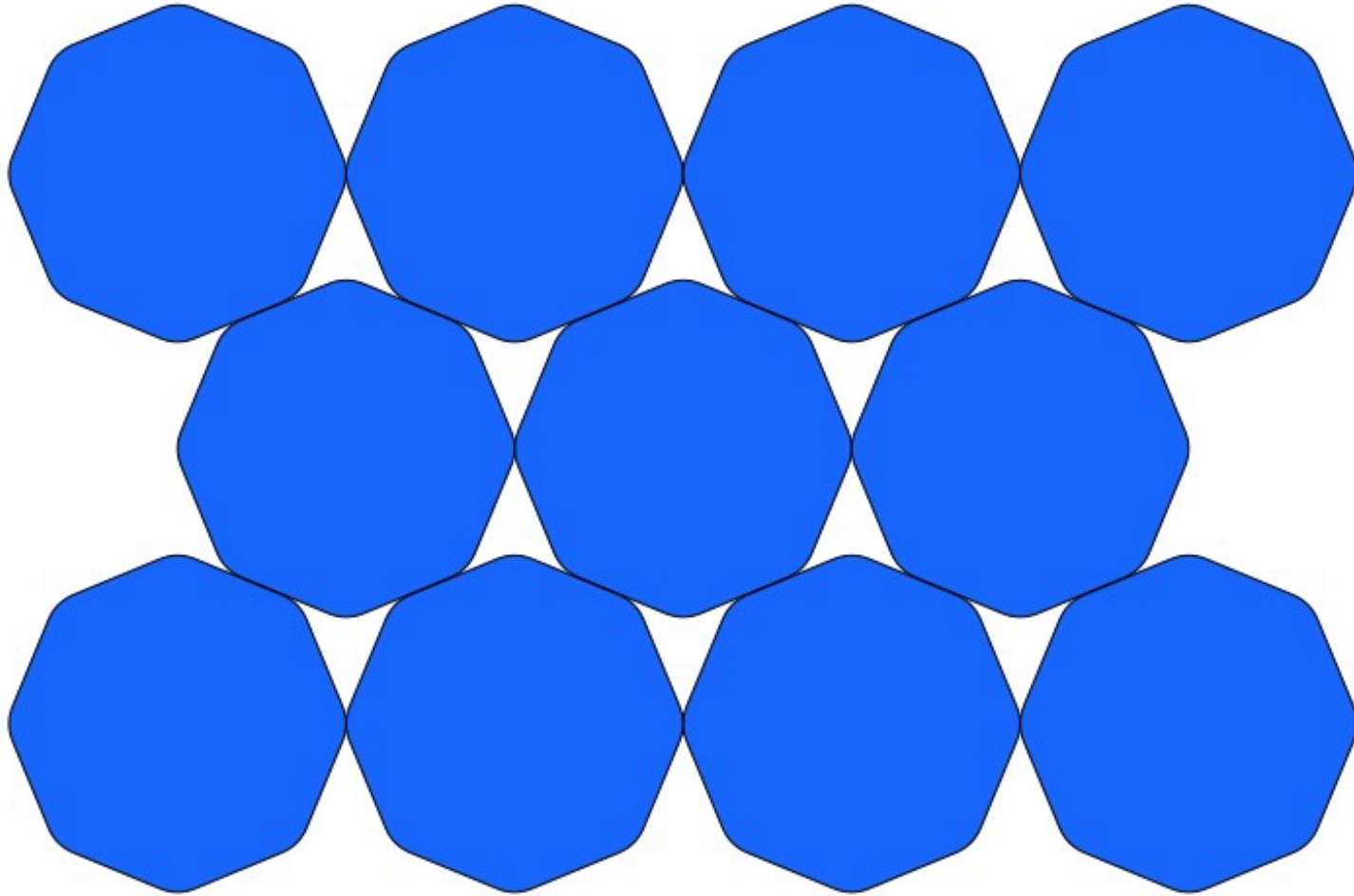
Why can we improve over circles?



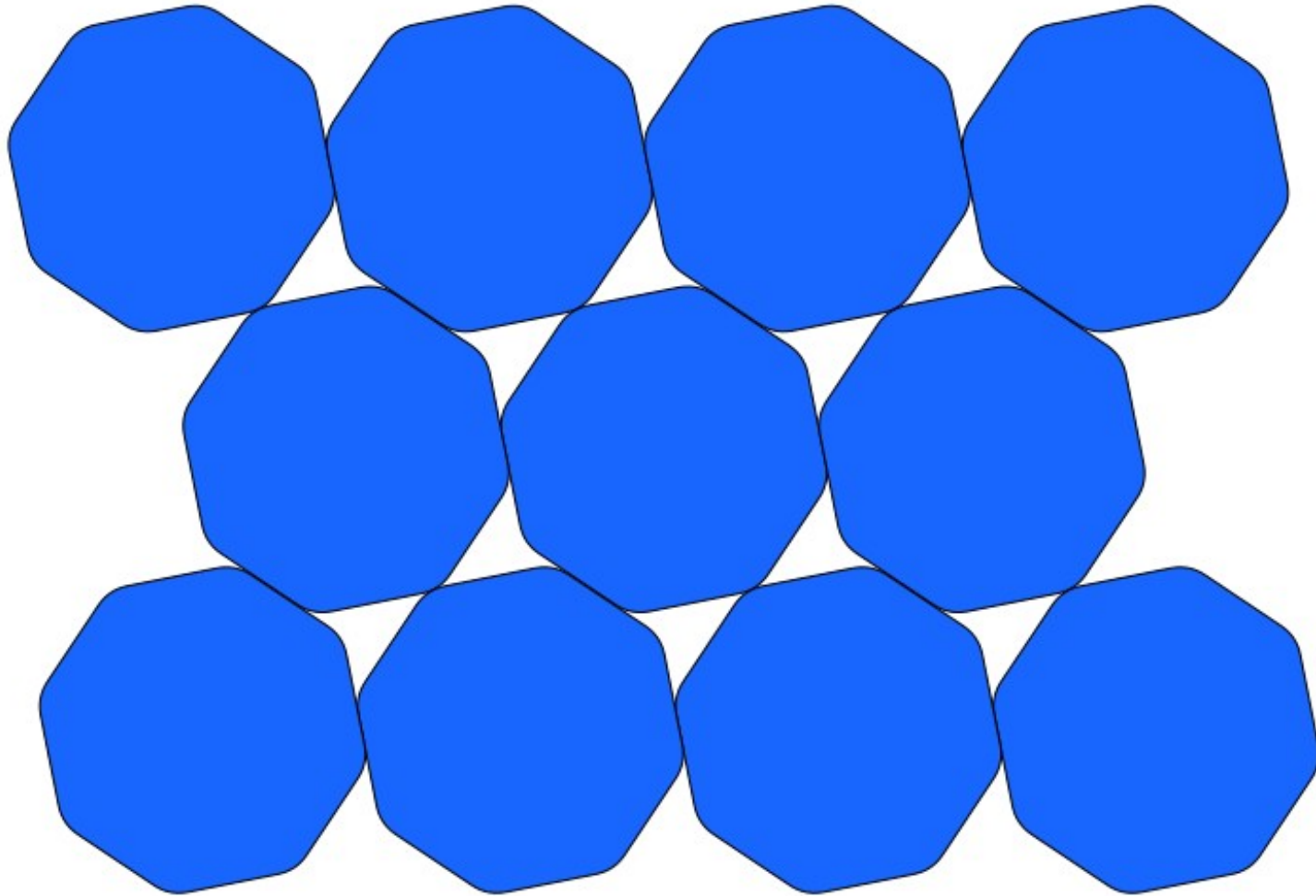
Why can we improve over circles?



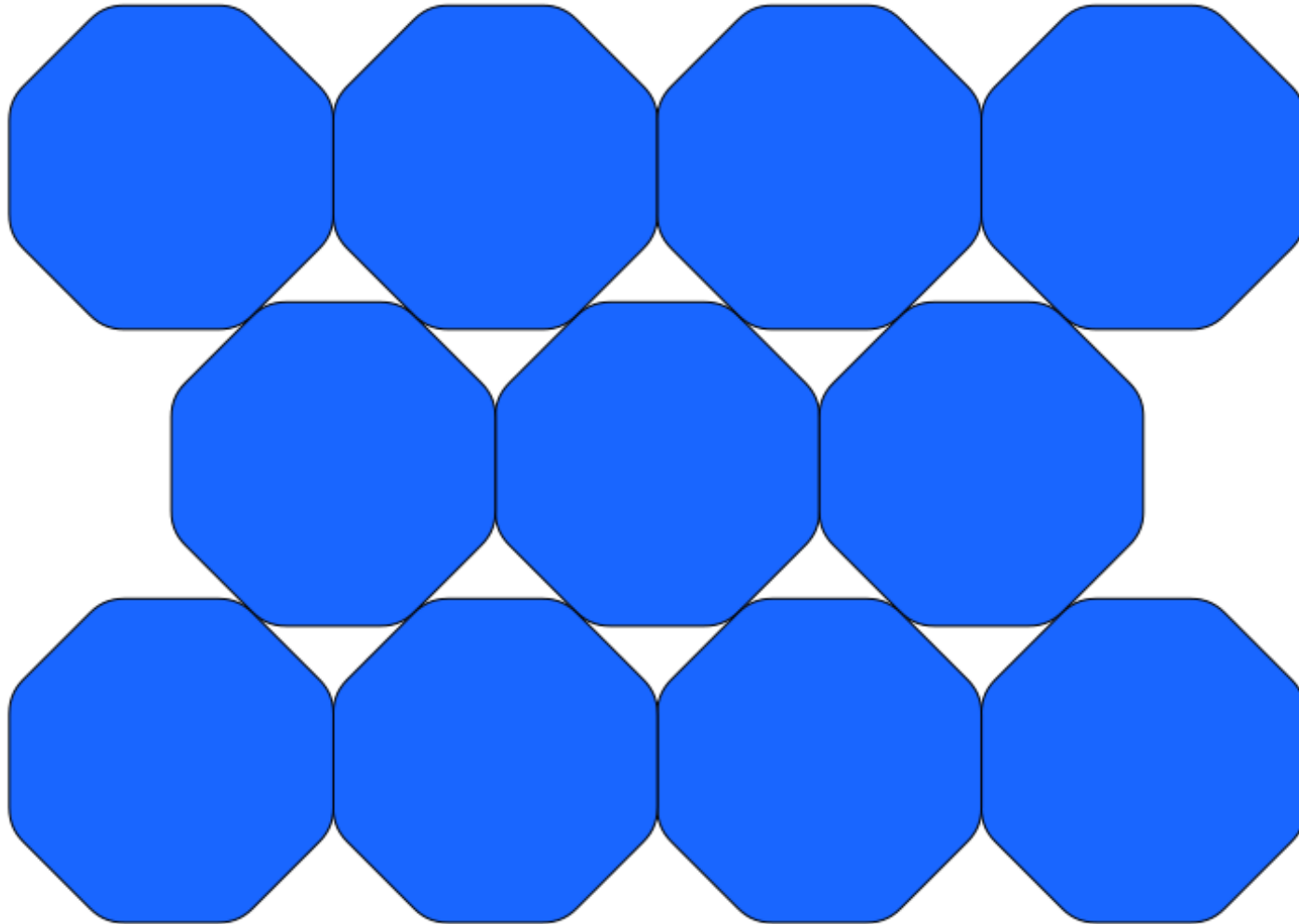
Why can we improve over circles?



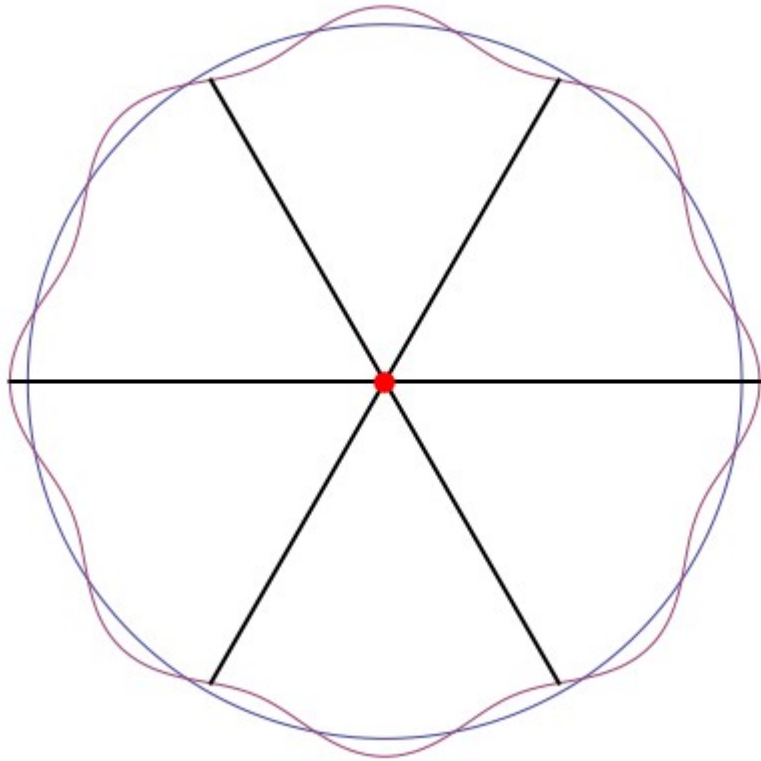
Why can we improve over circles?



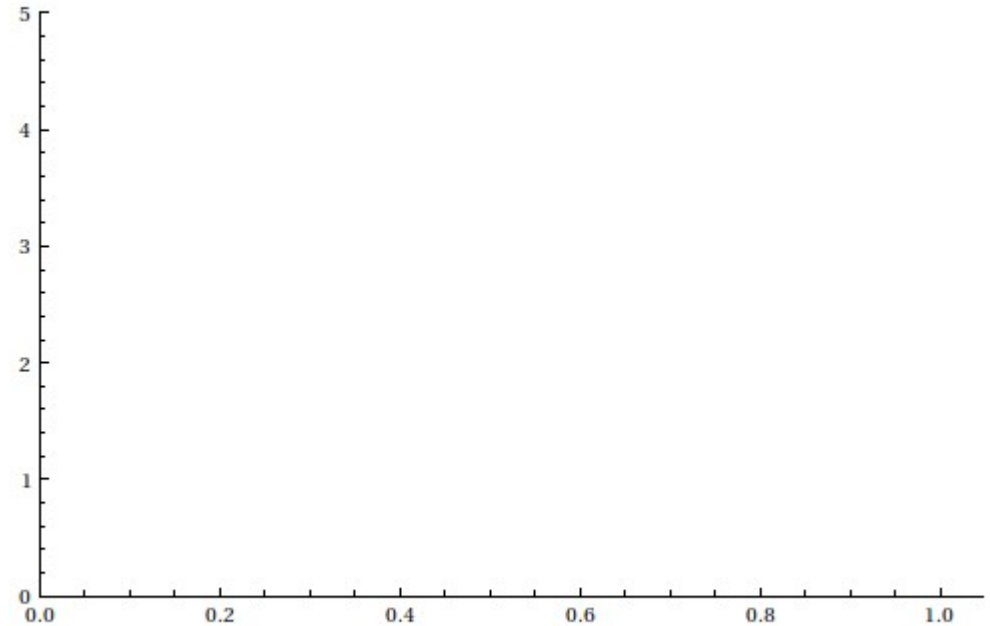
Why can we improve over circles?



Why can we improve over circles?

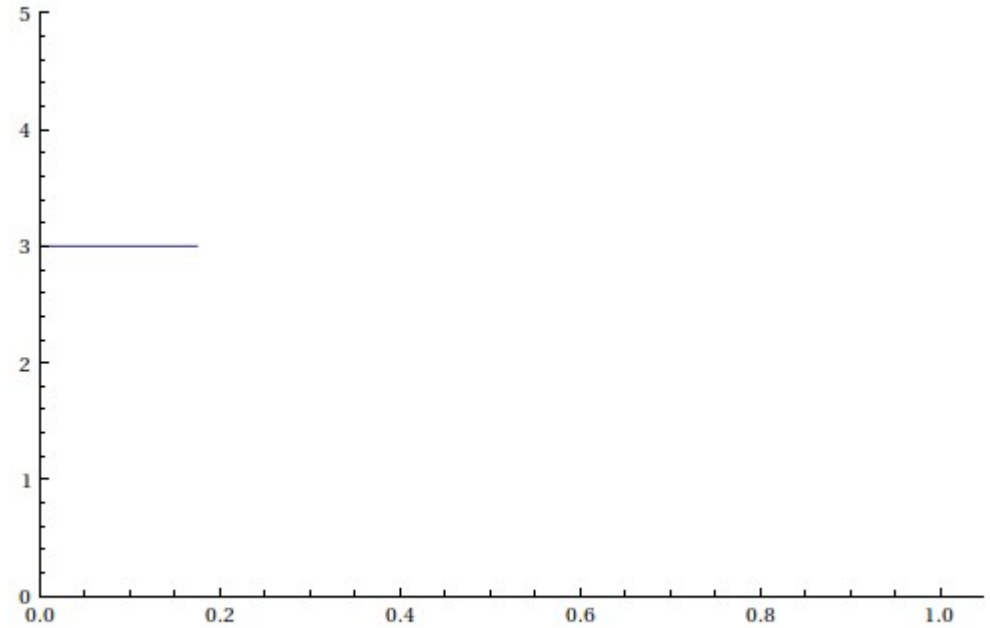
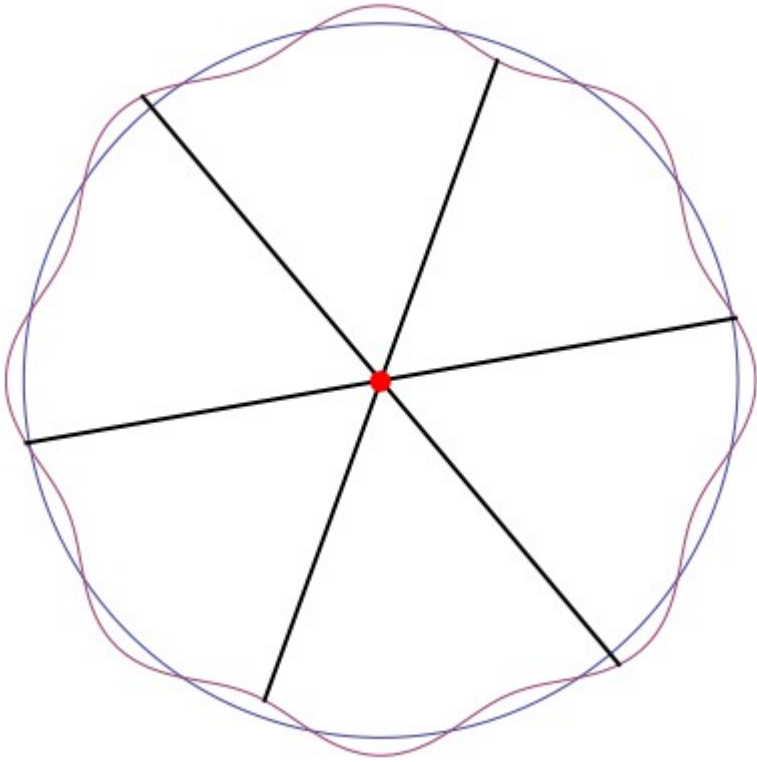


$$f(\theta)$$



$$\sum_{i=1}^6 f(\theta_i + \phi)$$

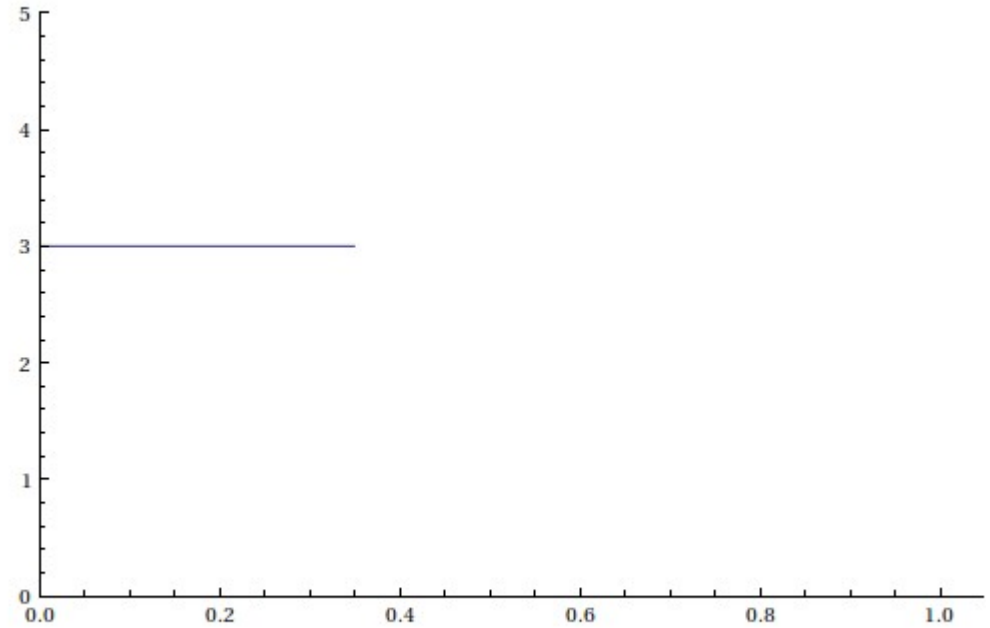
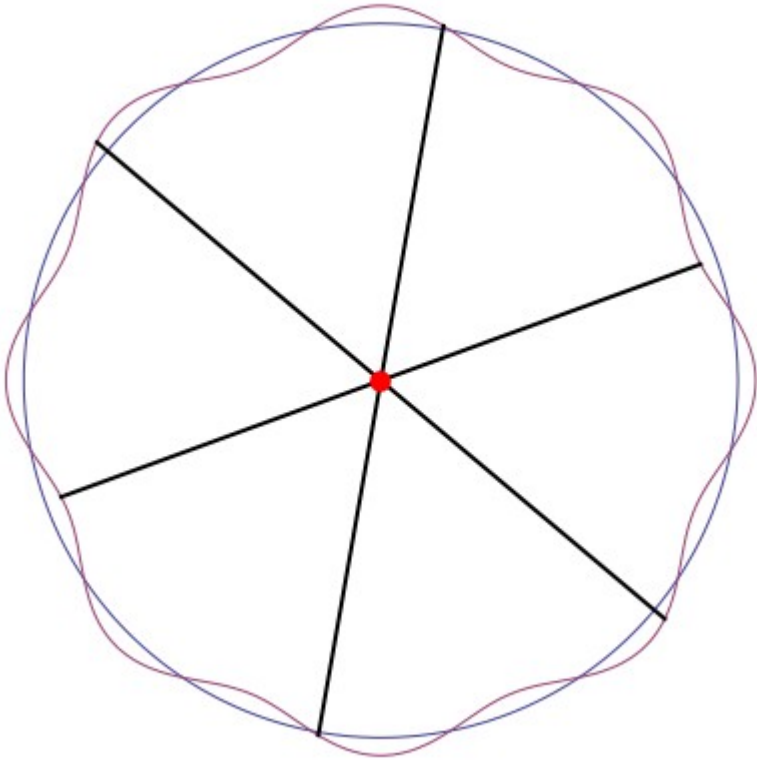
Why can we improve over circles?



$$f(\theta)$$

$$\sum_{i=1} f(\theta_i + \phi)$$

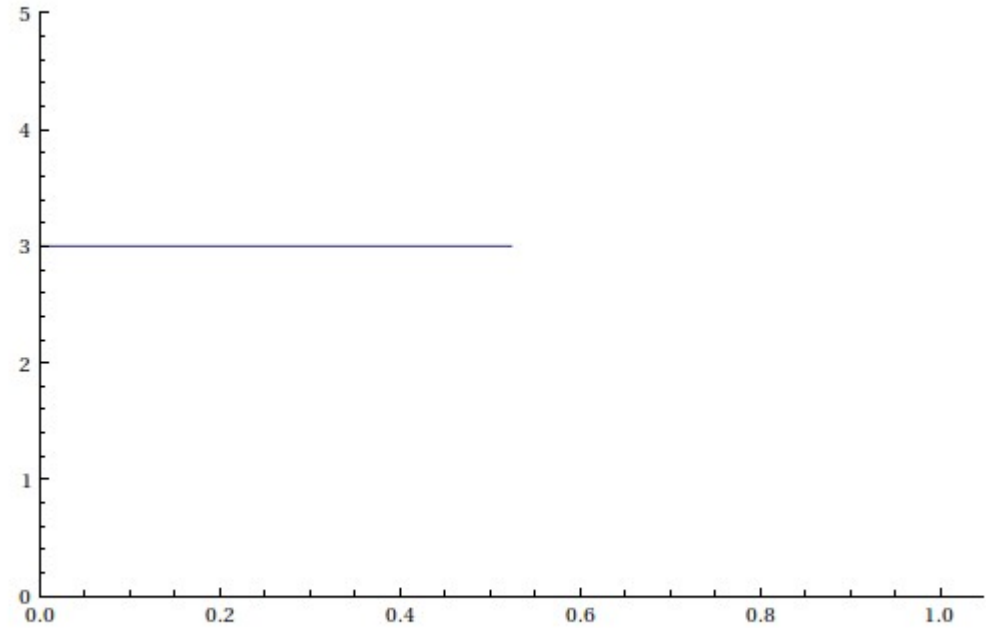
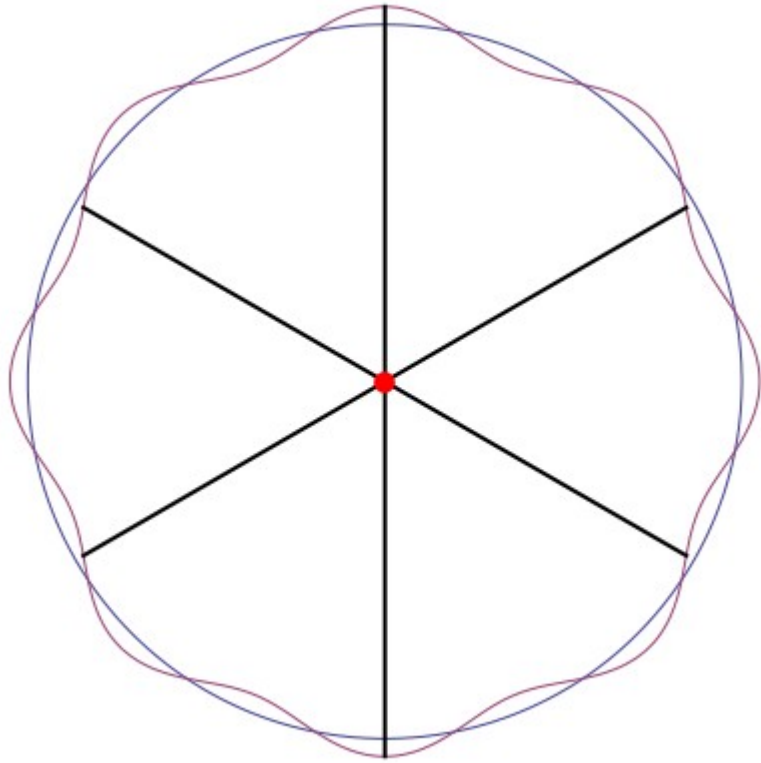
Why can we improve over circles?



$$f(\theta)$$

$$\sum_{i=1} f(\theta_i + \phi)$$

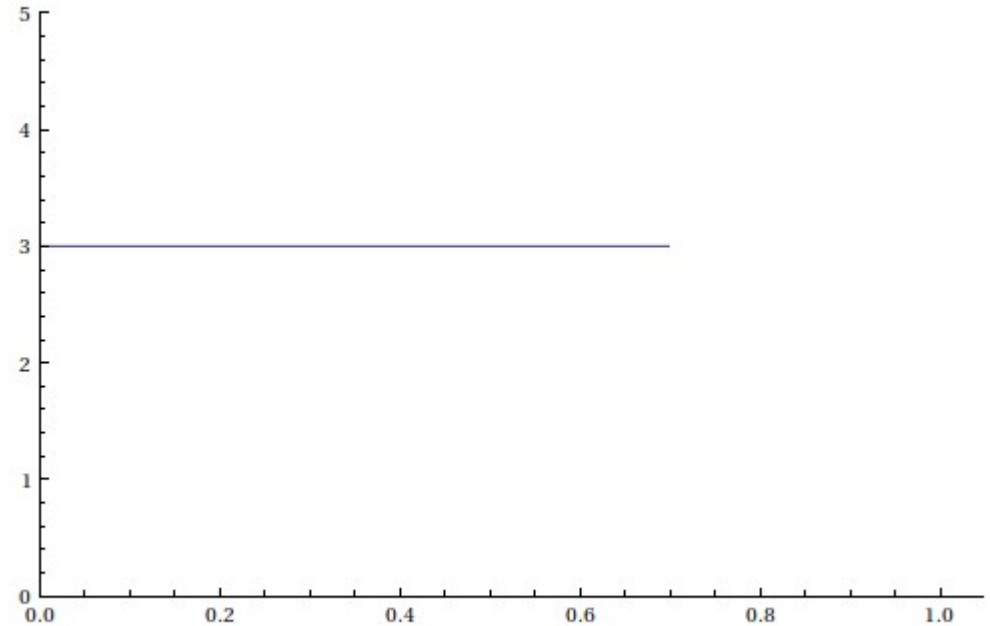
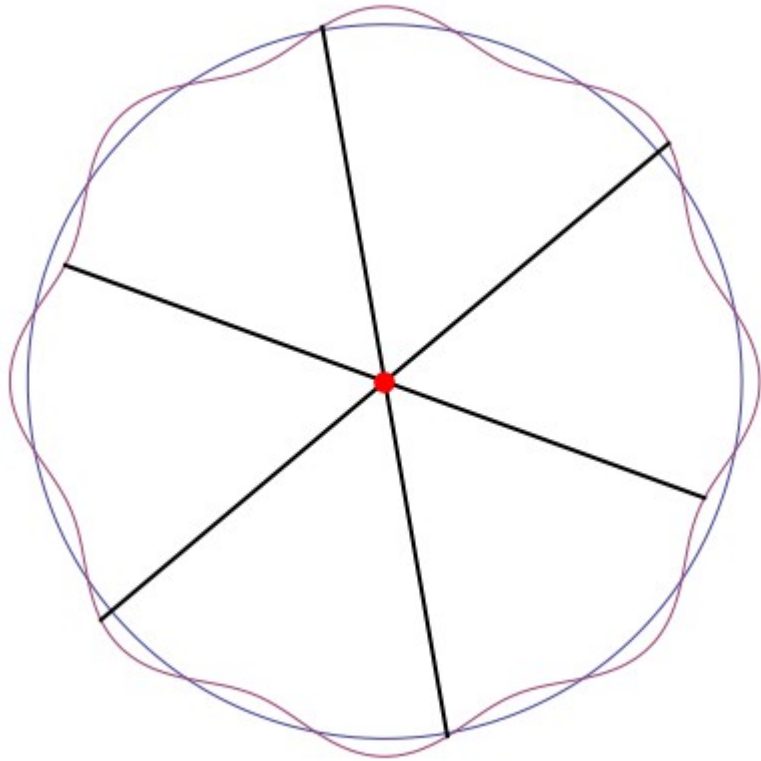
Why can we improve over circles?



$$f(\theta)$$

$$\sum_{i=1} f(\theta_i + \phi)$$

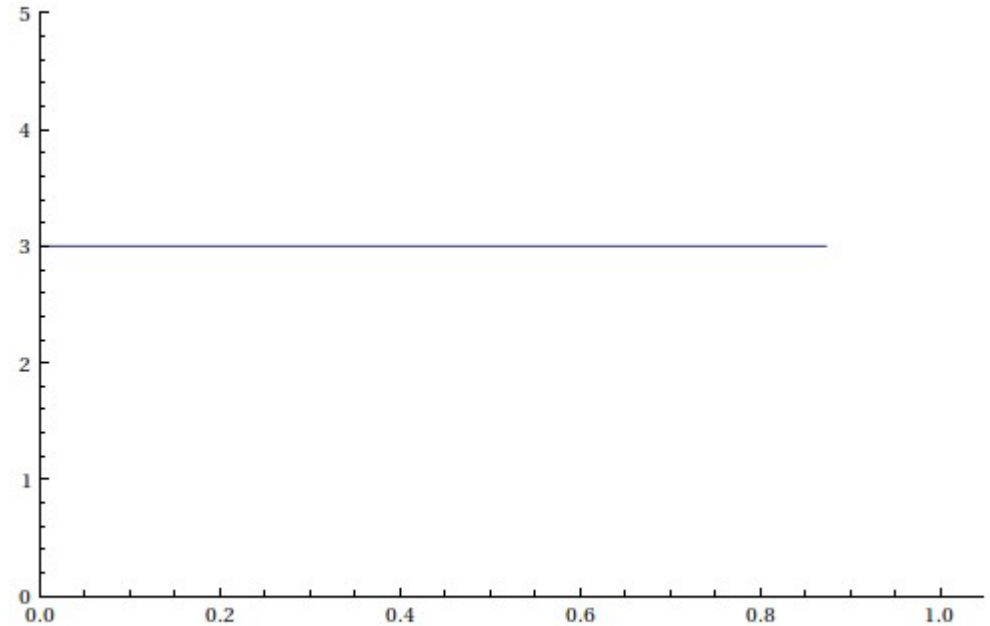
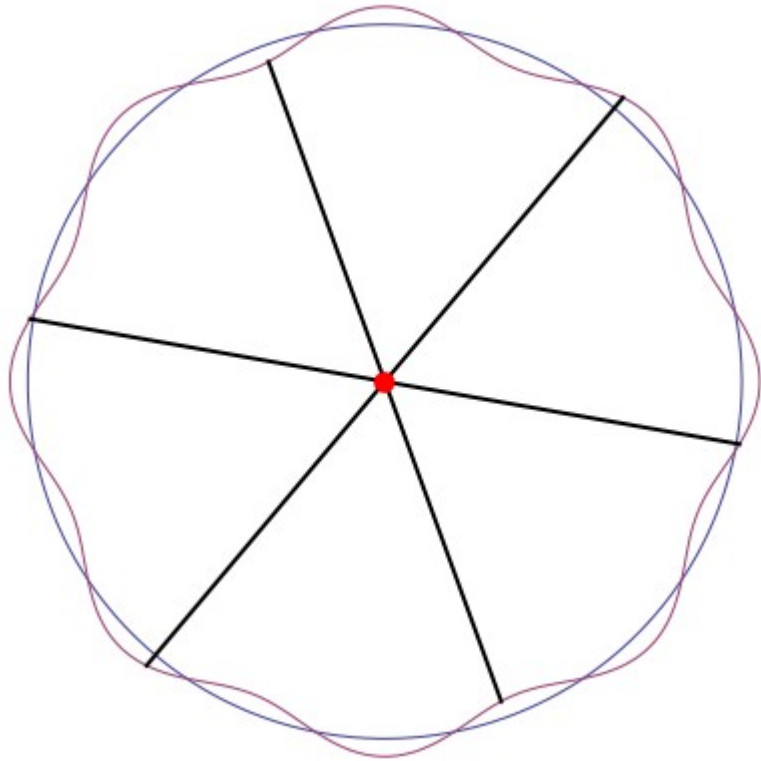
Why can we improve over circles?



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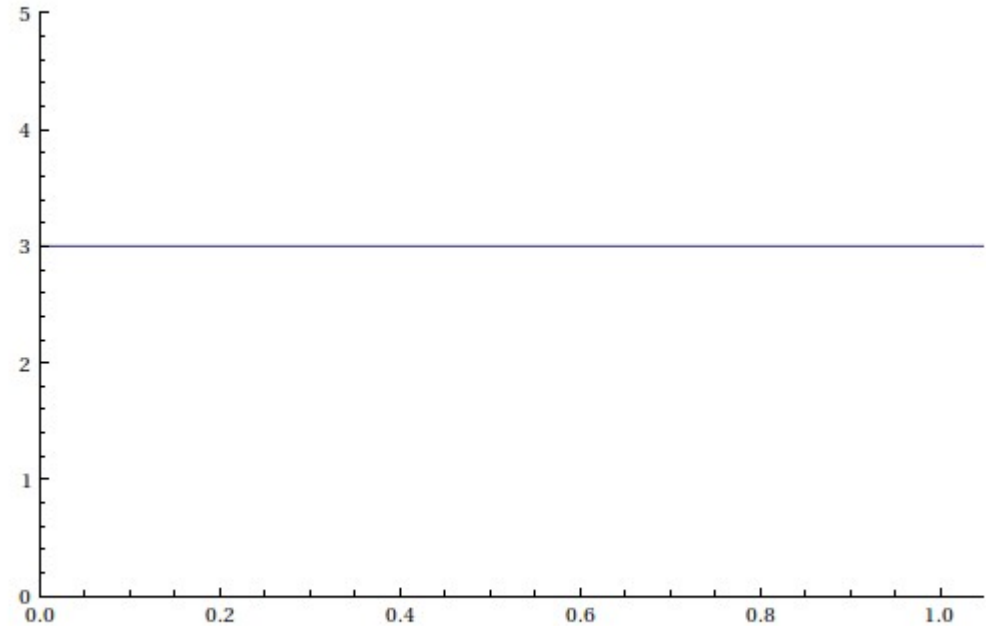
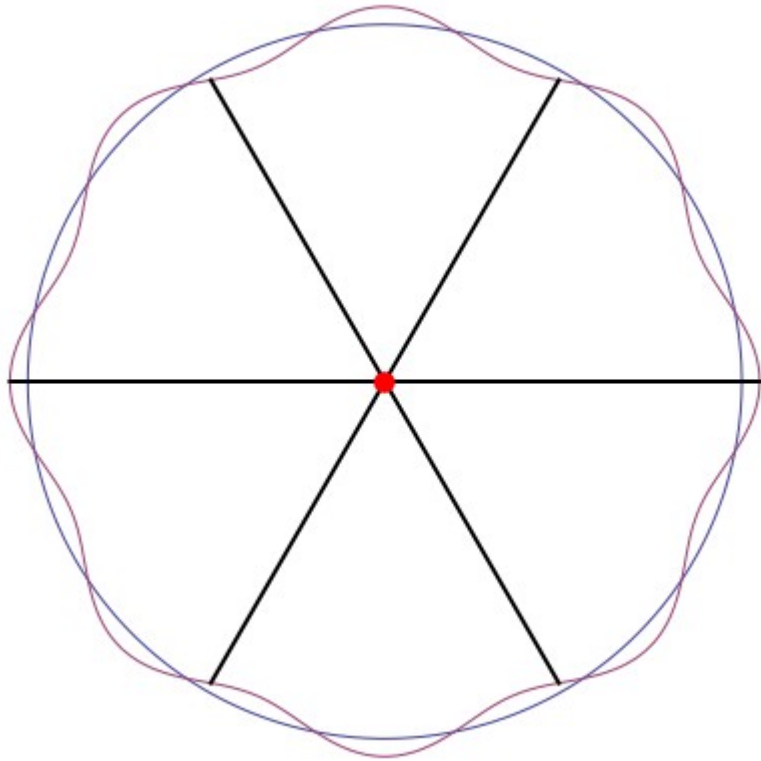
Why can we improve over circles?



$$f(\theta)$$

$$\sum_{i=1} f(\theta_i + \phi)$$

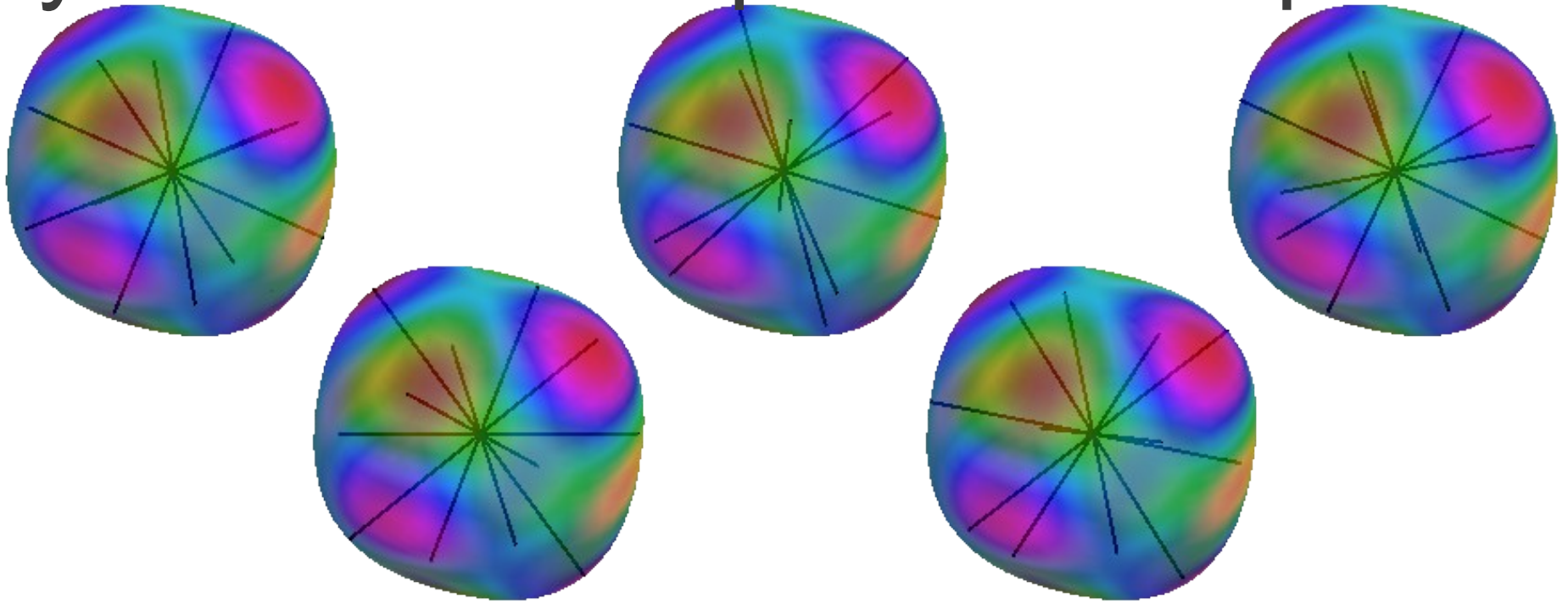
Why can we improve over circles?



$$f(\theta)$$

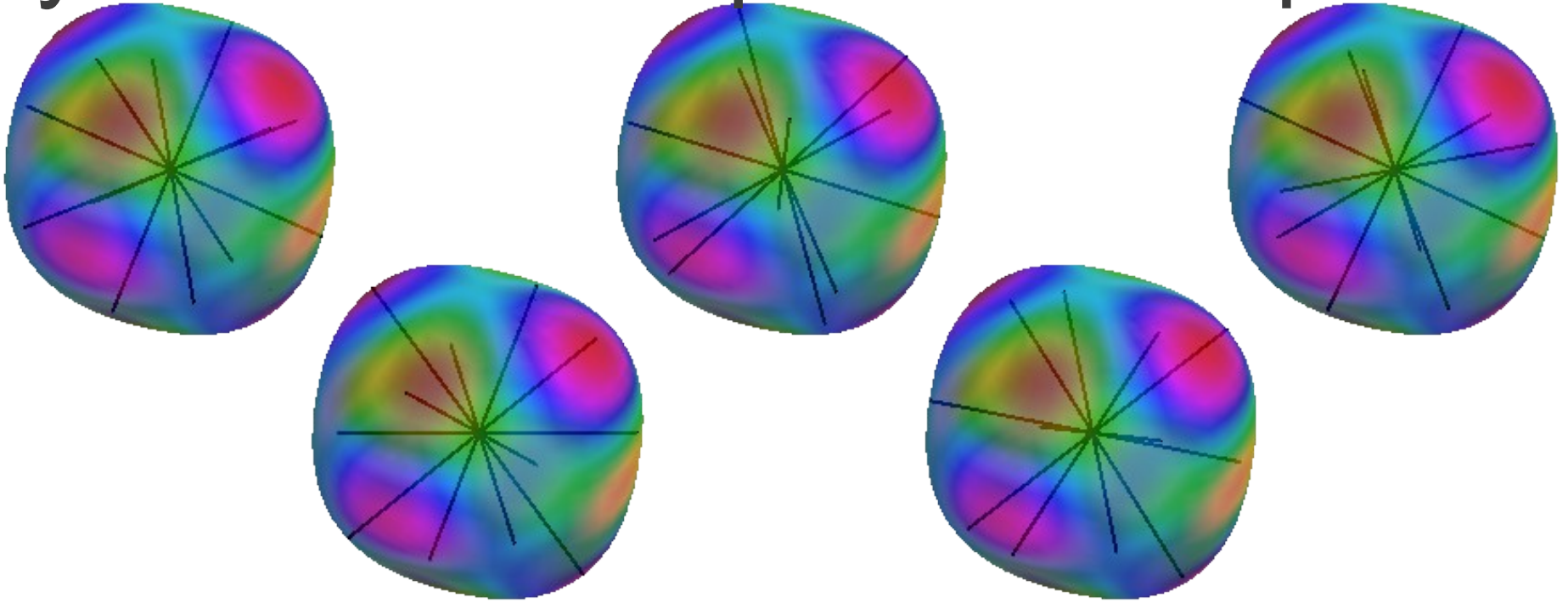
$$\sum_{i=1} f(\theta_i + \phi)$$

Why can we not improve over spheres?



$$\sum_{i=1}^{12} f(R\hat{u}_i) = \text{const.} \text{ if and only if}$$
$$f(u) = \text{monopole} + \text{dipole}$$

Why can we not improve over spheres?



Theorem (Kallus, Nazarov):
Sphere is locally worst among convex,
centrally-symmetric solids.

arXiv: 1212.2551

Higher dimensions, covering

d	2	3	4	5	6	7	8	24
L. worst packing		X						
L. worst covering	X	X						

arXiv: 1212.2551, 1301.5895