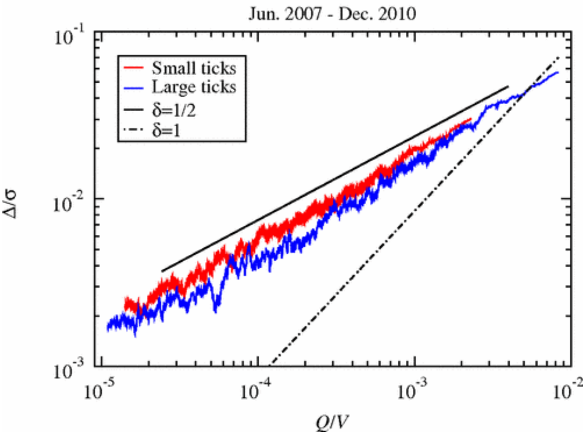
Soft gap in excitation spectrum of metastable states

Yoav Kallus

Santa Fe Institute

CCS 2015, Tempe
September 29, 2015

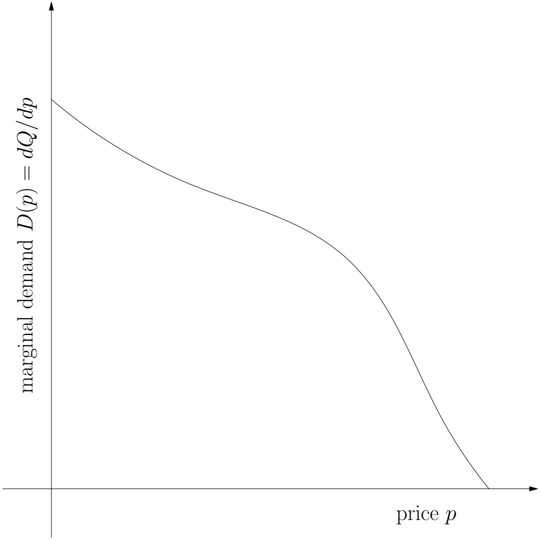
Anomalous price impact



Price impact from 5×10^5 trades on futures market by J.-P. Bouchaud's CFM (Tóth et al., PRX **1**, 021006 (2011)).

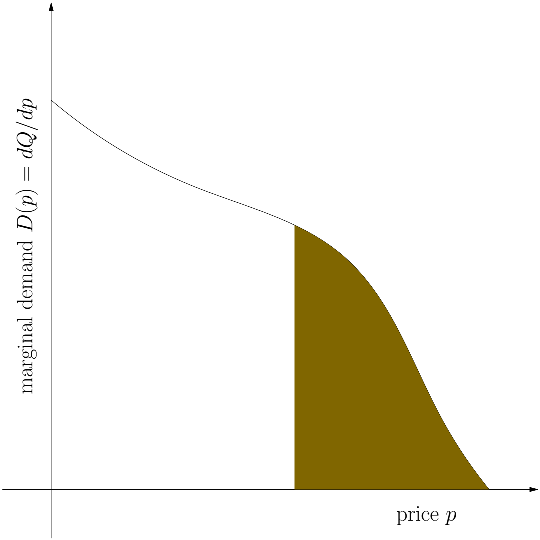
Why anomalous?

Consider a good with a fixed supply. How is its price determined?



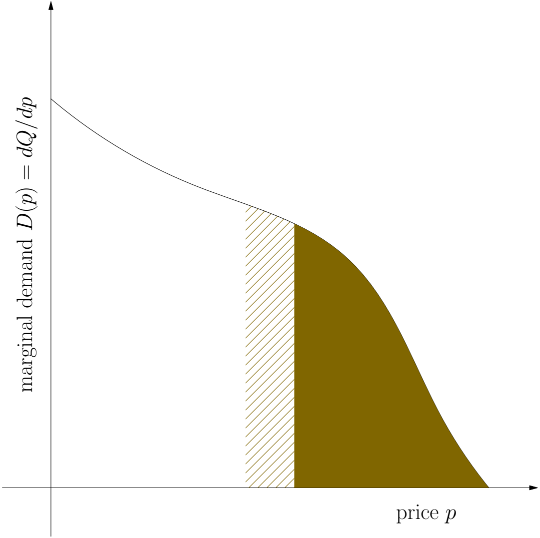
Why anomalous?

Consider a good with a fixed supply. How is its price determined?



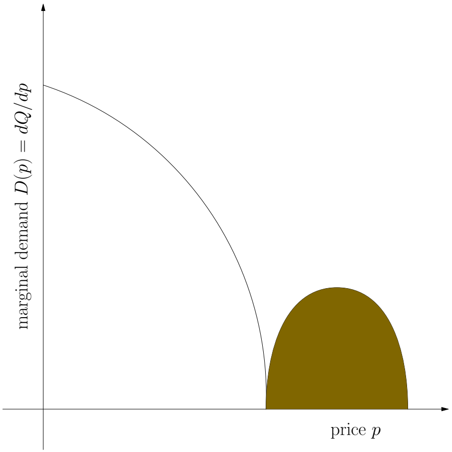
Why anomalous?

When supply changes, how does price change?

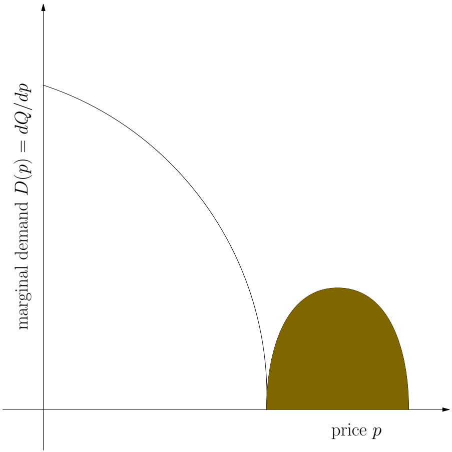


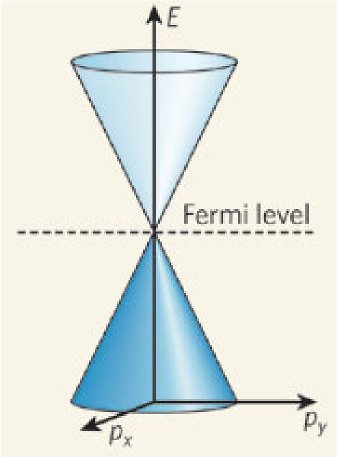
$$\Delta = -DQ + O(Q^2)$$

Vanishing liquidity



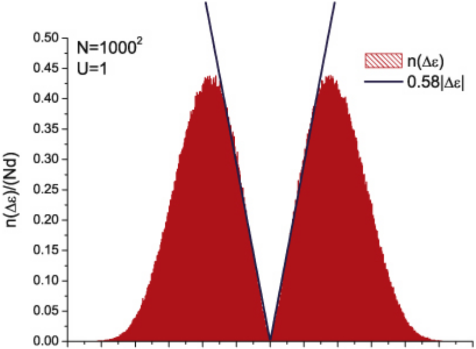
Vanishing liquidity





The Coulomb glass

$$H = \sum_i n_i u_i + \sum_{i,j} \frac{(n_i - \nu)(n_j - \nu)e^2}{r_{ij}}$$

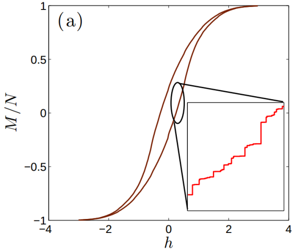


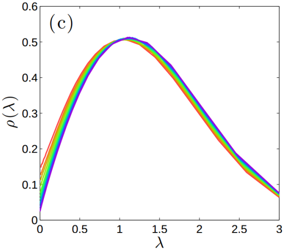
Gap appears at the Fermi level independent of filling

Pseudogap universality

Widely observed in disordered systems perched at metastable states

SK spin glass:

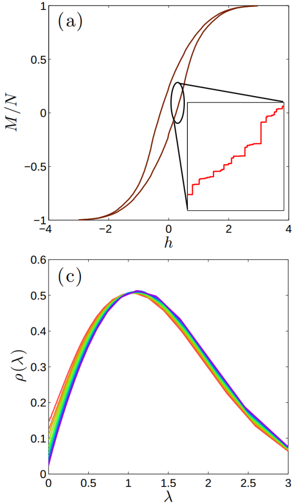




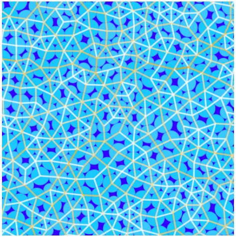
Pseudogap universality

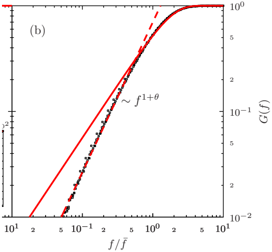
Widely observed in disordered systems perched at metastable states

SK spin glass:



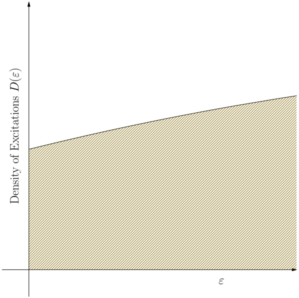
Random close packing:



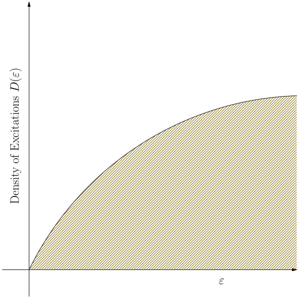


Critical stability

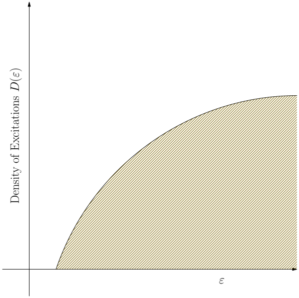
unstable (ungapped)



critical (pseudogap)

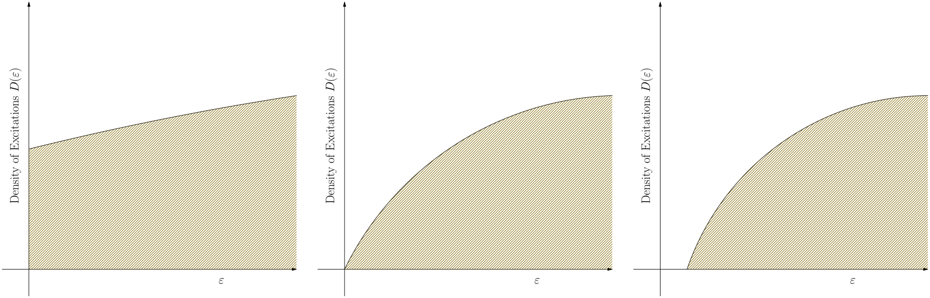


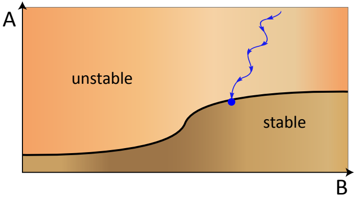
stable (gapped)



Critical stability

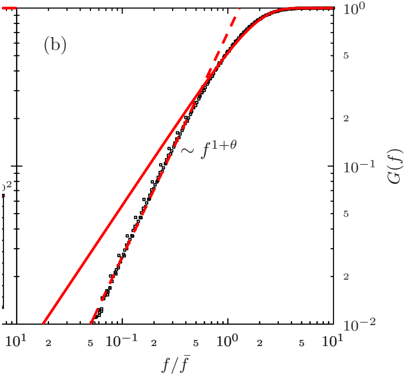
unstable (ungapped) | critical (pseudogap) | stable (gapped)

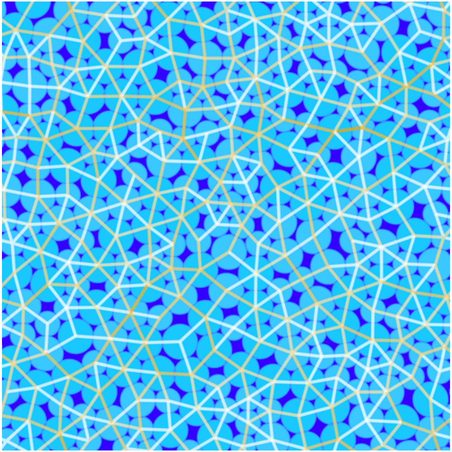




Random Close Packing

Force distribution:



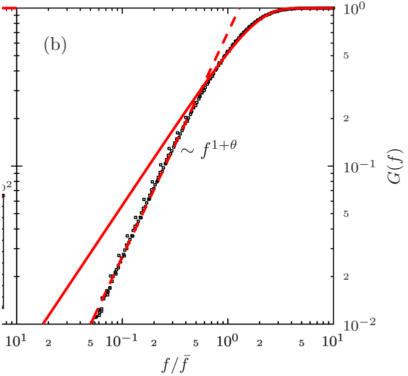


$$P(f) \sim f^{0.42}$$

Critical exponents independent of dimension.

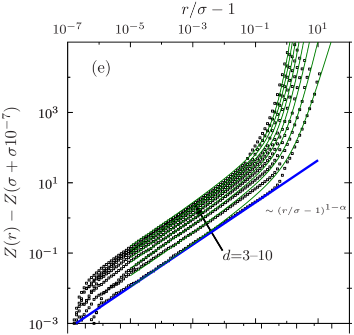
Random Close Packing

Force distribution:



$$P(f) \sim f^{0.42}$$

Gap distribution:

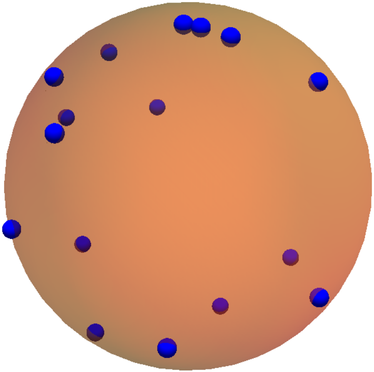


$$g(r) \sim r^{-0.42}$$

Critical exponents independent of dimension.

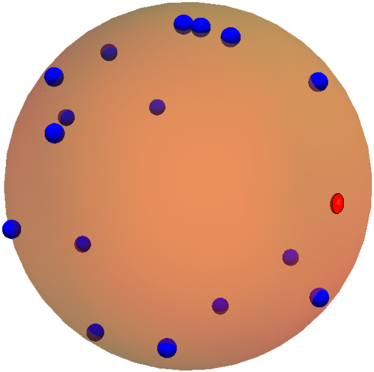
The "simplest model of jamming"

Place $m = \alpha n$ points randomly on the $(n-1)$-sphere, and try to find the point farthest from all of these.



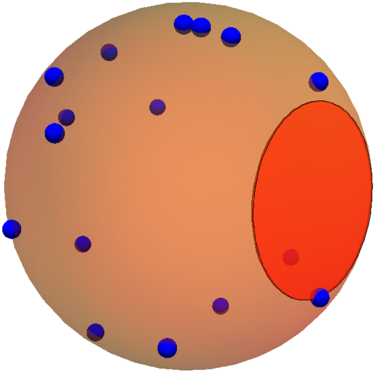
The "simplest model of jamming"

Place $m = \alpha n$ points randomly on the $(n-1)$-sphere, and try to find the point farthest from all of these.



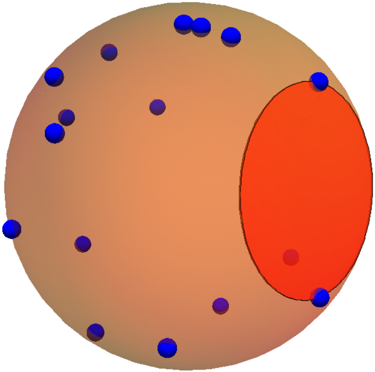
The "simplest model of jamming"

Place $m = \alpha n$ points randomly on the $(n-1)$-sphere, and try to find the point farthest from all of these.



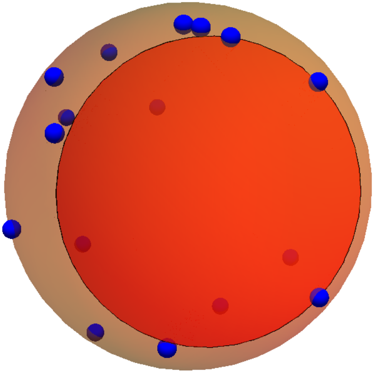
The "simplest model of jamming"

Place $m = \alpha n$ points randomly on the $(n-1)$-sphere, and try to find the point farthest from all of these.



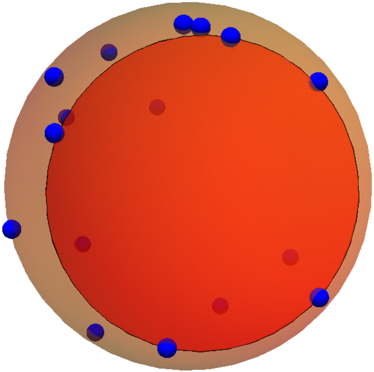
The “simplest model of jamming”

Place $m = \alpha n$ points randomly on the $(n-1)$-sphere, and try to find the point farthest from all of these.



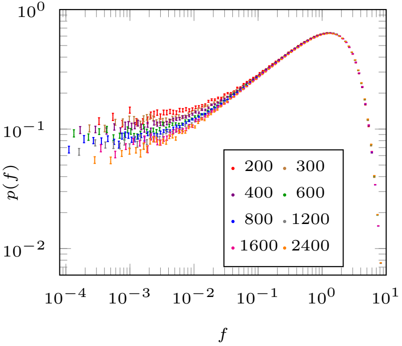
The "simplest model of jamming"

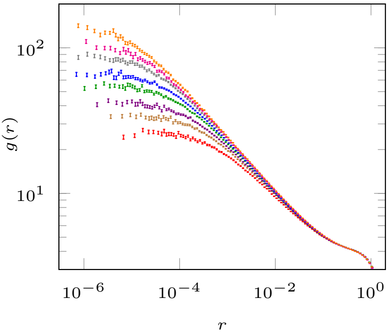
Place $m = \alpha n$ points randomly on the $(n-1)$-sphere, and try to find the point farthest from all of these.



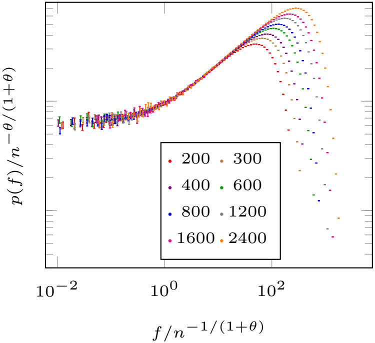
Same universality class as sphere packing in $d \to \infty$

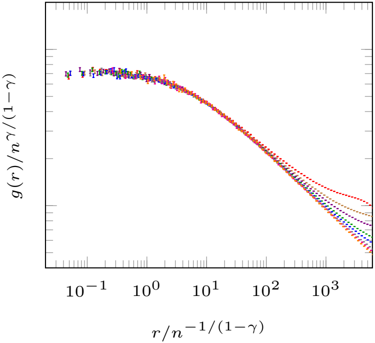
Numerical experiments





Numerical experiments

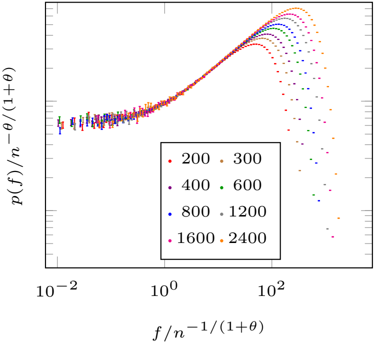


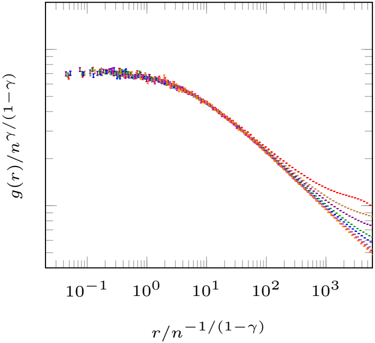


$$p(f) = n^{-\frac{\theta}{1+\theta}}\tilde{g}(rn^{\frac{1}{1+\theta}})$$

$$g(r) = n^{\frac{\gamma}{1-\gamma}}\tilde{p}(fn^{\frac{1}{1-\gamma}})$$

Numerical experiments





$$p(f) = n^{-\frac{\theta}{1+\theta}}\tilde{g}(rn^{\frac{1}{1+\theta}}) \qquad g(r) = n^{\frac{\gamma}{1-\gamma}}\tilde{p}(fn^{\frac{1}{1-\gamma}})$$

Future work: dynamics and avalanches